22127205

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 1

Thursday 3 May 2012 (afternoon)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	2	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

0120

17 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The same remainder is found when $2x^3 + kx^2 + 6x + 32$ and $x^4 - 6x^2 - k^2x + 9$ are divided by $x + 1$. Find the possible values of k.

2. *[Maximum mark: 5]*

Find the values of x for which the vectors $\begin{pmatrix} 1 \\ 2\cos x \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2\sin x \\ 1 \end{pmatrix}$ are perpendicular, $0 \le x \le \frac{\pi}{2}$.

3. *[Maximum mark: 5]*

On a particular day, the probability that it rains is $\frac{2}{5}$. The probability that the "Tigers" soccer team wins on a day when it rains is $\frac{2}{7}$ and the probability that they win on a day when it does not rain is $\frac{4}{7}$.

(a) Draw a tree diagram to represent these events and their outcomes. *[1 mark]*

(b) What is the probability that the "Tigers" soccer team wins? *[2 marks]*

(c) Given that the "Tigers" soccer team won, what is the probability that it rained on that day? *[2 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

4. *[Maximum mark: 5]*

(a) Expand and simplify $\left(x-\dfrac{2}{x}\right)^4$. *[3 marks]*

(b) Hence determine the constant term in the expansion $(2x^2+1)\left(x-\dfrac{2}{x}\right)^4$. *[2 marks]*

5. *[Maximum mark: 5]*

Three non-singular 2×2 matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{X}$ satisfy $4\boldsymbol{A}-5\boldsymbol{BX}=\boldsymbol{B}$.

(a) Find $\boldsymbol{X}$ in terms of $\boldsymbol{A}$ and $\boldsymbol{B}$. *[2 marks]*

(b) Given that $\boldsymbol{A}=2\boldsymbol{B}$, find $\boldsymbol{X}$. *[3 marks]*

6. *[Maximum mark: 7]*

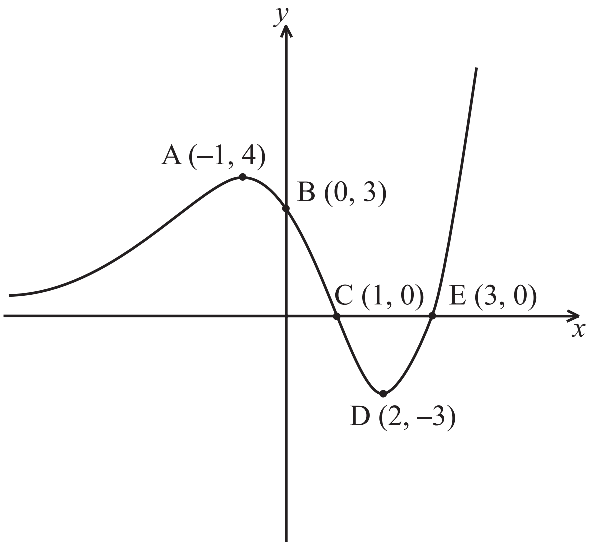
Given that $(4-5\mathrm{i})m+4n=16+15\mathrm{i}$, where $\mathrm{i}^2=-1$, find m and n if

(a) m and n are real numbers; *[3 marks]*

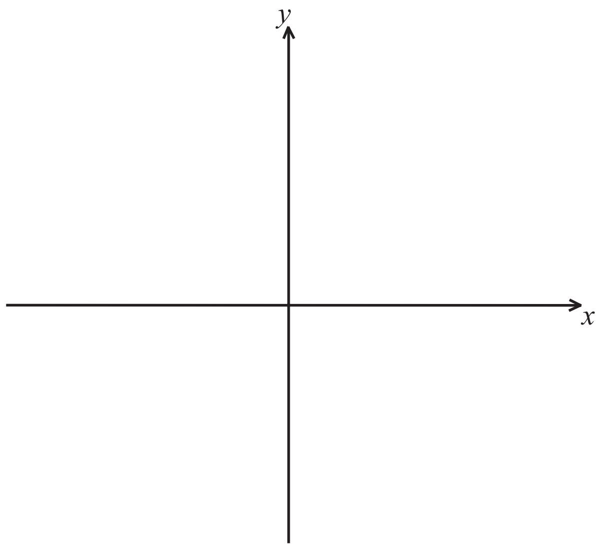
(b) m and n are conjugate complex numbers. *[4 marks]*

7. *[Maximum mark: 6]*

The graph of $y = f(x)$ is shown below, where A is a local maximum point and D is a local minimum point.



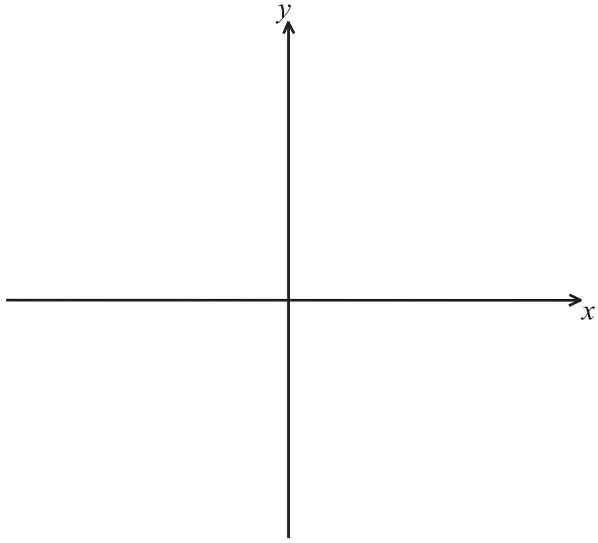
(a) On the axes below, sketch the graph of $y = \frac{1}{f(x)}$, clearly showing the coordinates of the images of the points A, B and D, labelling them A', B', and D' respectively, and the equations of any vertical asymptotes. *[3 marks]*



(This question continues on the following page)

(Question 7 continued)

(b) On the axes below, sketch the graph of the derivative $y = f'(x)$, clearly showing the coordinates of the images of the points A and D, labelling them A'' and D'' respectively. *[3 marks]*



8. *[Maximum mark: 6]*

Let $x^3 y = a\sin nx$. Using implicit differentiation, show that

$$x^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+6x^2\frac{\mathrm{d}y}{\mathrm{d}x}+(n^2x^2+6)\,xy=0.$$

9. *[Maximum mark: 6]*

Show that $\frac{\cos A + \sin A}{\cos A - \sin A} = \sec 2A + \tan 2A$.

10. *[Maximum mark: 9]*

The function f is defined on the domain $\left[0, \frac{3\pi}{2}\right]$ by $f(x) = e^{-x}\cos x$.

(a) State the two zeros of f. *[1 mark]*

. . . .

(b) Sketch the graph of f. *[1 mark]*

(This question continues on the following page)

(Question 10 continued)

(c) The region bounded by the graph, the x-axis and the y-axis is denoted by A and the region bounded by the graph and the x-axis is denoted by B. Show that the ratio of the area of A to the area of B is

$$\frac{e^{\pi}\left(e^{\frac{\pi}{2}}+1\right)}{e^{\pi}+1}.$$

[7 marks]

Please **do not** write on this page.

Answers written on this page will not be marked.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 18]*

Consider the following functions:

$$f(x) = \frac{2x^2+3}{75},\ x \ge 0$$

$$g(x) = \frac{|3x-4|}{10},\ x \in \mathbb{R}.$$

(a) State the range of f and of g. *[2 marks]*

(b) Find an expression for the composite function $f \circ g(x)$ in the form $\frac{ax^2+bx+c}{3750}$, where a, b and $c \in \mathbb{Z}$. *[4 marks]*

(c) (i) Find an expression for the inverse function $f^{-1}(x)$.

(ii) State the domain and range of f^{-1}. *[4 marks]*

The domains of f and g are now restricted to $\{0, 1, 2, 3, 4\}$.

(d) By considering the values of f and g on this new domain, determine which of f and g could be used to find a probability distribution for a discrete random variable X, stating your reasons clearly. *[6 marks]*

(e) Using this probability distribution, calculate the mean of X. *[2 marks]*

*Do **NOT** write solutions on this page.*

12. *[Total mark: 29]*

Part A *[Maximum mark: 12]*

(a) Given that $(x+\mathrm{i}y)^2 = -5+12\mathrm{i}$, x, $y \in \mathbb{R}$. Show that

(i) $x^2 - y^2 = -5$;

(ii) $xy = 6$. *[2 marks]*

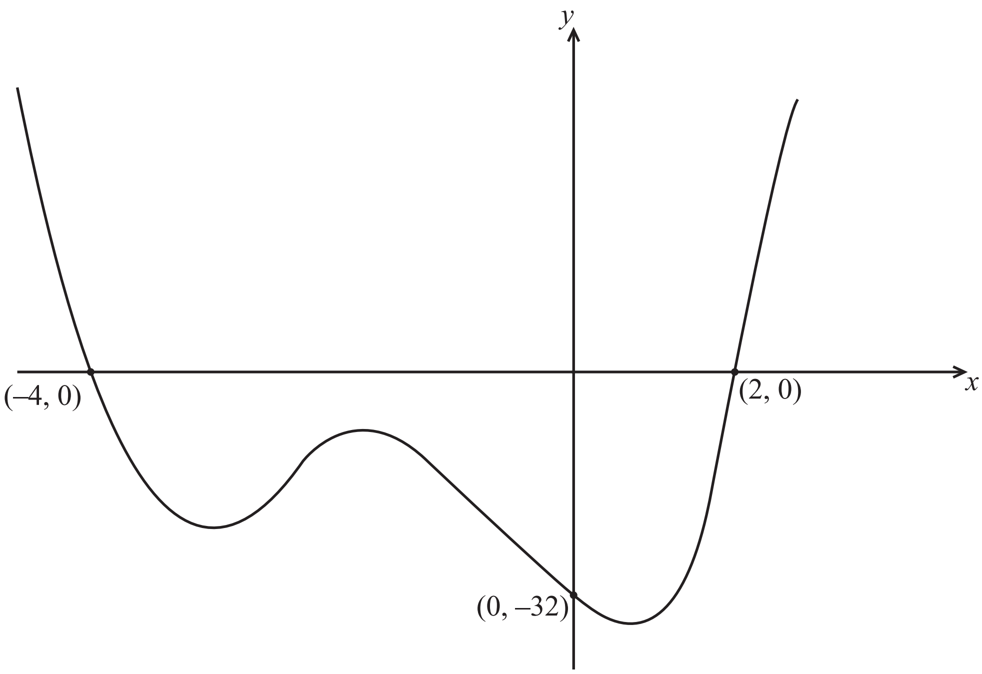
(b) Hence find the two square roots of $-5+12\mathrm{i}$. *[5 marks]*

(c) For any complex number z, show that $(z^*)^2 = (z^2)^*$. *[3 marks]*

(d) Hence write down the two square roots of $-5-12\mathrm{i}$. *[2 marks]*

Part B *[Maximum mark: 17]*

The graph of a polynomial function f of degree 4 is shown below.



(a) Explain why, of the four roots of the equation $f(x) = 0$, two are real and two are complex. *[2 marks]*

(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 12 continued)

(b) The curve passes through the point $(-1, -18)$. Find $f(x)$ in the form $f(x)=(x-a)(x-b)(x^2+cx+d)$, where $a, b, c, d \in \mathbb{Z}$. *[5 marks]*

(c) Find the two complex roots of the equation $f(x)=0$ in Cartesian form. *[2 marks]*

(d) Draw the four roots on the complex plane (the Argand diagram). *[2 marks]*

(e) Express each of the four roots of the equation in the form $r\mathrm{e}^{\mathrm{i}\theta}$. *[6 marks]*

13. *[Maximum mark: 13]*

(a) Using the definition of a derivative as $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$, show that the derivative of $\frac{1}{2x+1}$ is $\frac{-2}{(2x+1)^2}$. *[4 marks]*

(b) Prove by induction that the n^{th} derivative of $(2x+1)^{-1}$ is $(-1)^n\frac{2^n n!}{(2x+1)^{n+1}}$. *[9 marks]*

Please **do not** write on this page.

Answers written on this page will not be marked.

Please **do not** write on this page.

Answers written on this page will not be marked.

Please **do not** write on this page.

Answers written on this page will not be marked.